

Getting In Control

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And a Host of Colleagues & Students

Trajectory Control

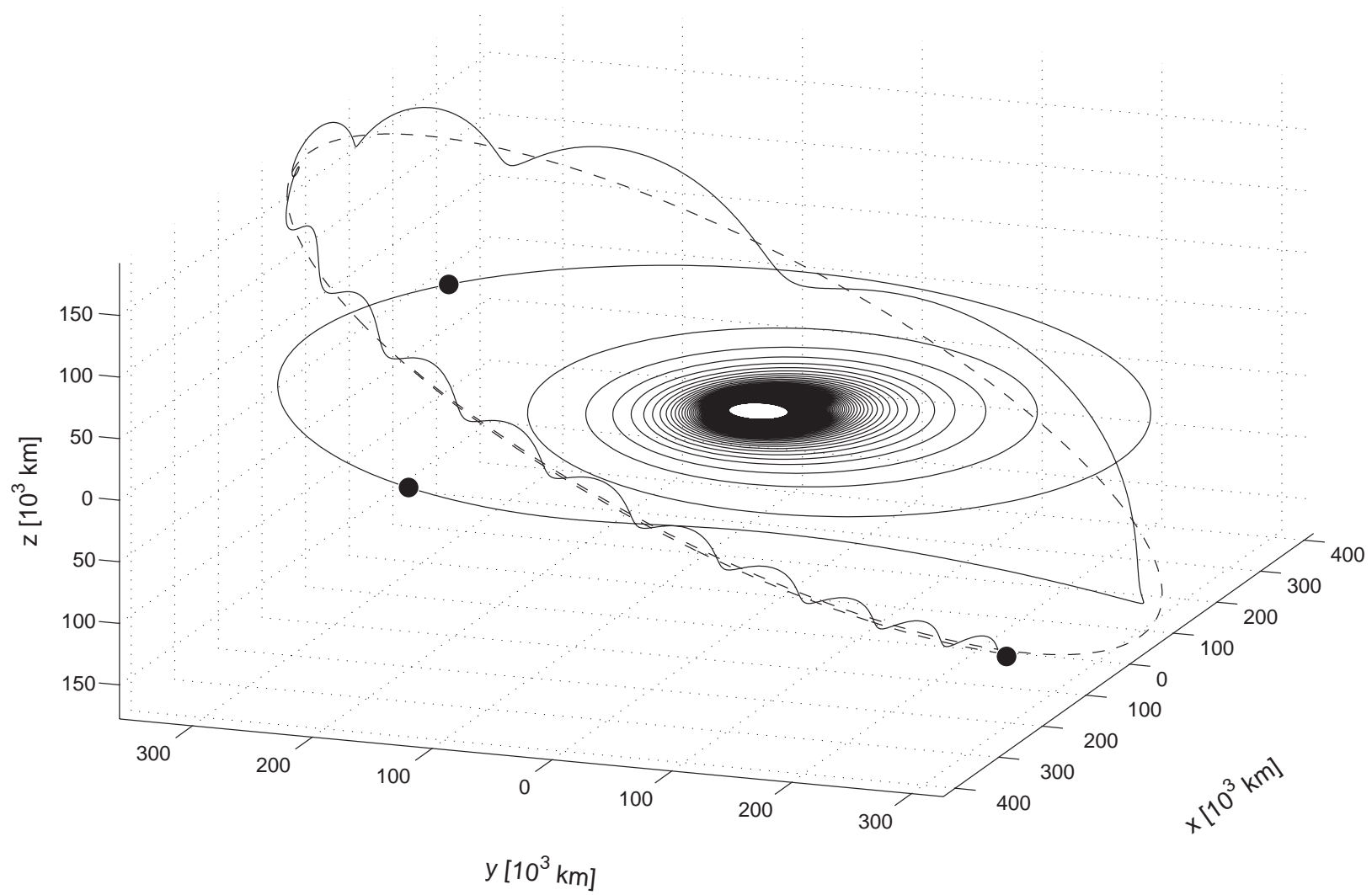
Given a system

$$\frac{dx}{dt} = f(x, u, t) \quad (1)$$

Find a control $u(t)$ that will steer $x(t)$ from point a to point b over a given time interval $[0, T]$.

Example: Have a low thrust space vehicle

- Start in earth orbit
- Wind up in specified moon orbit
- Actually an optimization problem but first
 - Have to be able to get close



Example 1:

$$\frac{dx}{dt} = x + u \quad (2a)$$

$$x(0) = 1 \quad x(5) = -4 \quad (2b)$$

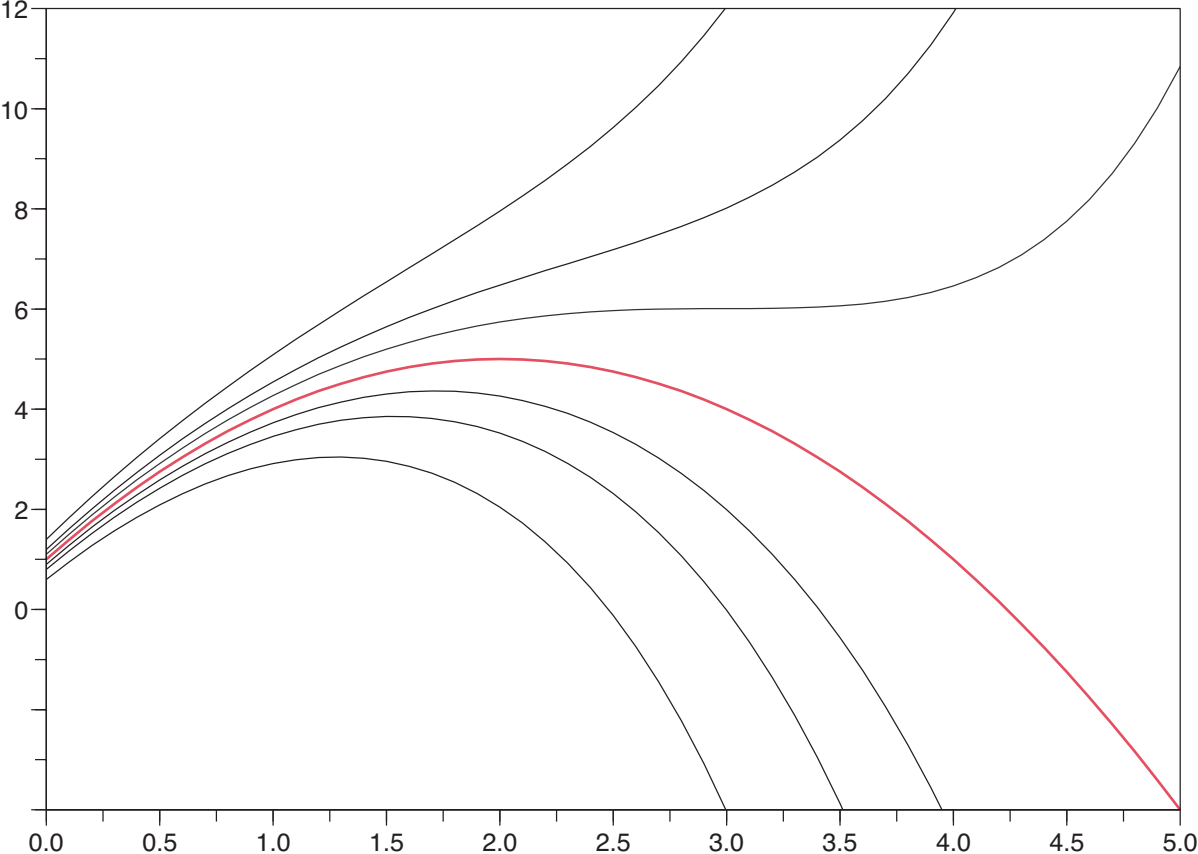
A control that works mathematically:

$$u = 3 + t^2 + 6t$$

- But if you run a simulation, it will not work
- If a real system it will not work

What is happening?

Several solutions for this u .



Math theory tells us

$$\boxed{\text{Solution}} = \boxed{\text{what we want}} + \boxed{\text{what it wants to do anyway}}$$

$$\boxed{\text{Solution}} = \boxed{\text{Forced Response}} + \boxed{\text{Free/Natural Response}}$$

This control is **open loop**

- Never looks at where you are actually at

Introduces important idea of **feedback**.

$$u = g(x) + v$$

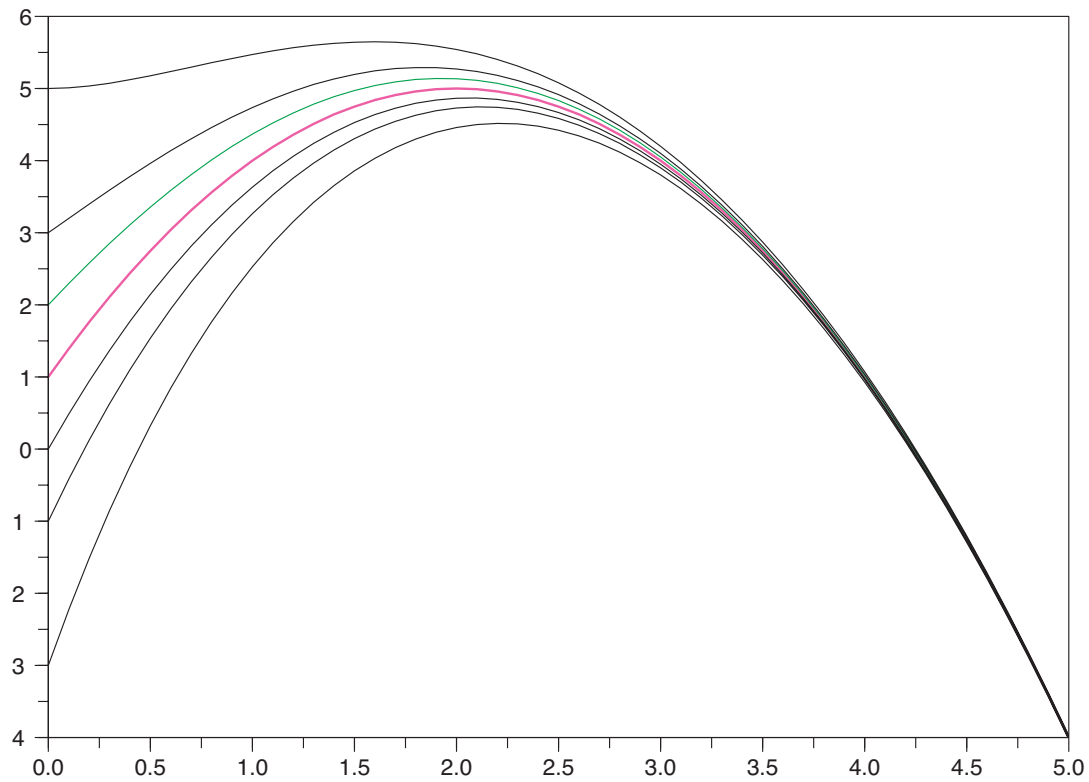
- $g(x)$ is feedback to make our system well behaved (**closed loop**)
- v then gets well behaved system to do what we want

Back to example

$$u = -2x + v$$

where $v = 5 + 2t - t^2$

New picture is



In control:

- Interested in all (or all nearby) solutions and how they act
- Want to be able to change the dynamics so they act correctly
- **KNOW WHAT THE SOLUTION SHOULD BE**
 - Change the equations to get that solution
 - Limitations on changes you can make