# MA 405. Introduction to Linear Algebra and Matrices

## Lecture details

Section 50| TuTh 11:45 - 1:00, SAS 1102 Instructor: S. Campbell | SAS 4124| slc@ncsu.edu| 919.515.3300 Office Hours: TuTh 2:00 - 3:00, and by appointment We do not use moodle: Instructor Web Page http://slc.math.ncsu.edu Course Web Page https://slc.math.ncsu.edu/COURSES/MA\_405\_19.html

## Course text

The textbook for this course is **Linear Algebra with Applications** by Steven J. Leon, Ninth Edition. It is possible that some additional material will be provided as the semester proceeds.

## Catalog Description

## Prerequisite: MA 241 (Co-requisite MA 242)

This course offers a rigorous treatment of linear algebra, including systems of linear equations, matrices, determinants, abstract vector spaces, bases, linear independence, spanning sets, linear transformations, eigenvalues and eigenvectors, similarity, inner product spaces, orthogonality and orthogonal bases, factorization of matrices. Compared with MA 305 Introductory Linear Algebra, more emphasis is placed on theory and proofs. MA 225 is recommended as a prerequisite. Credit is not allowed for both MA 305 and MA 405.

## **Course overview**

Linear Algebra provides one of the cornerstones for much of modern Mathematics, and has important applications in Physics, Engineering, and Economics. The main purpose of this course is to introduce the basic concepts from linear algebra, explain the underlying theory, the computational techniques, and study how these concepts and results can be productively used in other areas of mathematics and physical sciences, especially in applied mathematics where multivariable models are involved. Among the topics covered in this course will be: solving systems of linear equations using Gauss elimination, row echelon form, determinants, vector spaces, linear independence, bases, dimension, linear transformations, orthogonality, eigenvalues, and reduction of matrices to diagonal forms. If time permits, we will discuss applications of linear algebra to differential equations and/or quadratic forms. The subject involves a mixture of both the practical and the theoretical, and will provide in particular a good introduction to mathematical proofs. For this reason, the course is considered to be a difficult one in undergraduate mathematics, and the student should be prepared to invest considerable amount of time in understanding the class material and doing homework.

## Learning Objectives

Upon successful completion of this course, students will be able to:

- 1. Use Mathematical Notation and Terminology. The students will demonstrate mastery in using the mathematical notation and terminology of linear algebra. Students will read, interpret, and use the vocabulary, symbolism and basic definitions.
- 2. Understand and Describe the Fundamental Concepts of Linear Algebra. Students will identify and apply the theorems about abstract vector spaces and linear transformations; will gain a clear understanding of the basic concepts of linear algebra, such as linear independence of vectors, spanning sets, basis, similarity, eigenvalues and eigenvectors.
- 3. Identify and Utilize Linear Algebra Tools. The students will be able to apply course material along with techniques and procedures covered in this course to solve problems. Students will master techniques for solving linear systems by various matrix methods, compute the determinant and the inverse of a square matrix, compute various factorizations of matrices, apply the Gram-Schmidt process, calculate and analyze the characteristic equation of a matrix to determine its eigenvalues and eigenvectors. Moreover, students will apply properties and theorems about vector spaces to specific mathematical structures that satisfy the vector space axioms, will analyze the differences and similarities between spanning sets, bases, and orthogonal bases and will use the knowledge gained in this course to determine appropriate methods of proof for specific problems.
- 4. Develop Cognitive Skills. Students will demonstrate the ability to reason with abstract linear algebra concepts, to read and comprehend mathematical arguments utilizing direct and indirect proof, case analysis,

and mathematical induction. Students will develop familiarity with axiomatic approach in mathematics through the study of vector spaces and linear transformations. They will acquire a level of proficiency in manipulating linear algebra concepts, in analyzing and evaluating their applicability in their future studies, including graduate work, in academic areas requiring linear algebra as a prerequisite for work in occupational fields requiring a background in linear algebra.

## Grading Policy

The grading will be assigned on a 10-point scale: A: 90 – 100, B: 80 – 89, C: 70 – 79, D: 60 – 69, F: < 60

The cutoffs for the +/- grades are determined at the end of the semester. Your final grade in this course will be determined by marks earned on the final exam, three term tests, online homework assignments, and in-class homework. The weighting of these components are as follows:

Homework = 25 % Three term tests = 40 % Final Exam = 35 %

*Note:* I do NOT curve grades in this course. It is theoretically possible for everyone in the class to get an A (or an F). Your performance depends only on how you do, not on how everyone else in the class does. It is therefore in your best interests to help your classmates study and do HW not to be turned in, while keeping the academic integrity policy in mind.

#### Term Tests 40%

There will be three closed book, closed notes in-class term tests. Dates will be on the course web page. *No re-tests* will be given. If you miss a test because of an undocumented or unexcused absence, a zero will be entered for that test grade. Students who are unable to take the test at those times (with a documented excuse that they cannot, and not just that they don't want to) will schedule an alternate time to take the exam. This should be scheduled as soon as possible after the exam, ideally within a week. No test will be dropped but the lowest of your three term tests will be counted for half as much. So the three term tests will count 16%, 16%, and 8%.

#### Final Exam 35%

The final exam is mandatory, cumulative and will be held in the usual classroom. Date on the course section web page.

Homework Assignments Homework will consist of three types.

- 1. Some HW will be assigned from the text. This is just for practice. The answers are in the back of the book. This HW is not turned in. You can discuss this work with others in class including the TA.
- 2. Some HW from either the book or assigned in class will be to be turned in. This will be graded and is to be your own work. This can only be discussed with the instructor or the TA. Unless stated otherwise it is due two class period after it is assigned. We shall refer to this as HWS.
- 3. There are also problems on Webwork. This is best done prior to the exams but is due by midnight before the final exam. This will be referred to as HWW.

The homework part of the grade is based on HWS and HWW. The exact weighting will be decided once we see how much HWW there is. Each will count for at least 40% of the homework score. HWW problems are generally 1 or 2 pts each but a few are more. HWS will vary.

#### Corrections to the grading

If you believe an error has been made in grading on a test write a statement making your case and bring it to your instructor. I will give partial credit to a partial[y correct solution that was neatly presented and does not include a fatal error. What constitutes a fatal error will be discussed during the semester. You have 1 week after the test is returned to request re-grading. Do not alter the original work!

#### Test Make-Up Policy

All *anticipated absences* must be excused in advance of the test date. These include university duties or trips (certified by an appropriate faculty or staff member), required court attendance (certified by the Clerk of Court),

or religious observances (certified by the Department of Parent and Family Services 515-2441). *Emergency absences* must be reported as soon as possible once returning to class and must be appropriately documented (illness by an attending physician or family emergencies by Parent and Family Services). If you are sick on a test day and decide not to come to class, go to the health center or other medical facility. Students who miss a test and have a university-approved excuse must submit appropriate documentation.

<u>Attendance</u> is expected every day as it is critical for the understanding of the material and not attending class serves as its own penalty because this material takes much longer to learn independently. You are responsible for keeping up with missed work so that you do not fall behind. Office hours will not be utilized to re-teach material presented in class.

**<u>FIRST ASSIGNMENT</u>** Your first assignment is to get the name and email address of at least one student in class. If you miss a class it is your responsibility to get the class notes and find out what was said in class. At some point during the semester having this contact person will often be necessary.

#### Instructor's commitment

You can expect your instructor to be courteous, punctual, well organized, and prepared for lecture and other class activities; to answer questions clearly and in a non-negative fashion; to be available during office hours or to notify you beforehand if they are unable to keep them; to provide a suitable guest lecturer when they are traveling; and to grade uniformly and consistently according to the posted guidelines.

<u>The Math Multimedia Center</u> is a tutorial center for undergraduate students that need help in their mathematics courses (100- through 300-level), and is staffed by math graduate students familiar with the material taught in these courses.

Location: SAS Hall 2103/2105

Hours: Monday - Friday 8:00 am - 5:00 pm

You can also get help with your courses (not only math) at the NCSU Undergraduate Tutorial Center.

#### Add/Drop Regulation

Undergraduate students are expected to complete all courses for which they are enrolled as of census date (the official enrollment date defined as the 10th day of fall and spring terms and the 3rd day of summer terms). Undergraduate course drops after census date will now be considered to be course withdrawals and will result in W grades on the transcript. Undergraduates will be limited to a <u>maximum of 16 hours of course withdrawals</u> after census date and before the drop date October 13, 2017 for their entire undergraduate career at NC State. These course withdrawals will count as attempted hours for course repeat, financial aid satisfactory academic progress, and tuition surcharge calculations.

#### Students with disabilities

"Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, students must register with Disability Services for Students at 1900 Student Health Center, Campus Box 7509, 515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (REG02.20.1)." All accommodations are done at the disability center and are arranged by the student.

#### **Student Evaluations**

Online class evaluations will be available for students to complete during the last three weeks of classes. You will receive an email message directing you to a website where you can login using your Unity ID and complete the evaluation. All evaluations are confidential; instructors will not know how any one student responded to any question, and students will not know the ratings for any instructors.

#### Academic Integrity Statement and Academic Dishonesty

Both faculty and students at North Carolina State University have a responsibility to maintain academic integrity. An informational brochure about academic integrity is available from the university and students are encouraged to obtain a copy.

"Academic dishonesty is the giving, taking, or presenting of information or material by a student that unethically or fraudulently aids oneself or another on any work which is to be considered in the determination of a grade or the completion of academic requirements or the enhancement of that student's record or academic career." (NCSU Code of Student Conduct) Scholarly activity is marked by honesty, fairness and rigor. A scholar does not take credit for the work of others, does not take unfair advantage of others, and does not perform acts that frustrate the scholarly efforts of others. The violation of any of these principles is academic dishonesty. Penalties for a violation: For the first violation, you will receive a zero for your work and be put on academic integrity probation for the remainder of your stay at NCSU. The second violation may result in your suspension from NCSU. Both situations will involve the Office of Student Conduct.

## **Other Remarks**

- You are more than welcome to visit me during my official office hours, but I am available at many other times (just before class is usually not a good time, however). Please make an appointment if necessary.
- A good way to contact your instructor is by email. Please make sure that you include your name and the course number and section in the subject line of your email.
- Your email address registered with the NCSU online directory will be used for announcements associated with this class. It is your responsibility to maintain a valid email address and check/empty your Inbox regularly.
- Keep all your quizzes and tests for future reference. Do not discard until after you have received your final grade and you do not have any questions about your grade.
- Please check the course webpage regularly, as it will be continuously updated with announcements, any changes in the schedule, homework problems, solutions, review sheets, and other additional course materials.
- Please mark the test dates on your calendar and do not set your dental/doctor/interview... appointments on top of the test dates.
- Be respectful to your peers and to your instructor. All cell phones should be turned off during class and no eating, drinking, or any tobacco products are allowed in the classroom. Please leave your laptop in your bag during class time. Students who do not follow these guidelines may be asked to leave class.
- Brimmed hats, other than baseball caps with the brim turned to the back may not be worn to exams. The exception is documented medical or religious reasons.

#### MA405 Tentative Schedule

Some lectures may last more than one class period and some less. We may deviate from the order a few times. Any such deviations will be pointed out in class. The course is self contained but it is assumed that the student has added vectors, seen matrices, and seen a system of equations before. The numbers in parenthesis are sections in Leon.

- Lecture 1. Matrices and Systems of Equations (1.1-1.6)
- Lecture 2. Gaussian Elimination
- Lectures 3. Determinants (2.1, 2.2)
- Lecture 4. Vector Spaces. Examples. Subspaces (3.1,3.2)
- Lecture 5. Linear Independence, Basis, coordinates, Dimension (3.3, 3.4)
- Lecture 6. Change of Basis, coordinates (3.5)
- Lecture 7. Row space, Column space (3.6)
- Lectures 8 Null Space/ Rank Theorem.
- Lecture 9. Linear Transformations, range, kernel (4.1)
- Lectures 10. Matrix representation (4.2)
- Lecture 11. Similarity (4.3), isomorphism
- Lecture 12. Norms Supplemental material
- Lecture 13. Scalar product in  $\mathbb{R}^n$ . Orthogonal subspaces (5.2).
- Lecture 14. Inner product Spaces (5.4)
- Lecture 15. Orthonormal sets (5.5)
- Lecture 16. Gram-Schmidt orthogonalization and examples (5.6, 5.7)
- Lectures 17. Eigenvalues & Eigenvectors (6.1)
- Lectures 18. Systems of Differential Equations (6.2)
- Lectures 19.. Eigenspaces, Diagonalization. Similarity (6.3)
- Lecture 20. Hermitian matrices (6.4)
- Lecture 21. Singular value decomposition (6.5)
- Lecture 22. Quadratic forms (6.6)
- Lecture 23. Positive definite and nonnegative matrices (6.7, 6.8)
- Lectures 24. Orthogonal Complements. Least squares approximation. SVD (A mix of 1.1, 6.5)
- Time permitting we might discuss the Jordan form at the end of the semester.

## Good Luck!