On a New Predictive Control Strategy: Application to a Flexible-Joint Robot

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Abstract

It is shown that it is possible to greatly improve the performance of the predictive control strategy proposed by Jankowski and Van Brussel [9] by using preliminary linear feedback. This is illustrated using the example of a flexible-joint robot path following problem.

1. Introduction

A predictive control strategy for the prescribed path following control of flexible joint-robots has recently been proposed by Jankowski and Van Brussel [9]. Their method uses efficient numerical techniques for integrating higher index differential algebraic equations (DAE's) for constructing the control. In [5], we presented an analysis of this control strategy and showed that by a simple extension of this strategy (in particular by adding a preliminary feedback) it can be applied to a much broader class of problems. In this paper, we show that it was a coincidence that the Jankowski and Van Brussel strategy worked on the particular flexible-joint robot considered in [9] and explain their observation that their controller becomes unstable for small sampling periods. We then show that by adding a preliminary feedback, their control strategy works for any (sufficiently small) sampling period and has better performance.

In Section 2, we present the path following problem and the predictive control strategy proposed by Jankowski and Van Brussel. In Section 3, we review the analysis of this strategy, introduced in [5, 6], which shows how preliminary feedback may be needed in certain circumstances. Finally, we give a comparative study of the performance of the controller without and with preliminary feedback for different sampling periods, using the example of a flexible-joint robot introduced in [9].

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2. Predictive Control of Jankowski and Van Brussel

The solution to tracking problems can be found in nonlinear control theory as exact input-output linearization [8]. Using this method, as long as the original system is minimum-phase, it is possible to construct a stabilizing controller which achieves asymptotic tracking for any sufficiently differentiable reference trajectory. The main drawback with this approach is its computational complexity. Even for small nonlinear systems, the symbolic computation required for exact input-output linearization can be prohibitive. In order to overcome this difficulty Jankowski and Van Brussel [9] have proposed a new design methodology, closely related to predictive control or receding horizon control. As in all these strategies their approach is in part open-loop and in part closed-loop. They measure the state, compute an open-loop control which is applied on a short time period, and start over. To compute the open-loop control they "minimize" in some sense the tracking error using numerical integration schemes for higher index DAE models [2]. As with many other predictive controller approaches another advantage is that it is easy to incorporate changing constraints. In our case, one merely adds to, or alters, the equations being fed to the DAE integrator Nonpredictive approaches typically require a complete recomputation of the nonlinear control law.

The path following control problem considered hre is the following. Given a system of differential equations

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i = f(x) + G(x)u \tag{2.1a}$$

and the output vector function

$$y = h(x) \tag{2.1b}$$

the objective is to find a stabilizing feedback control $u(t,x)$, such that the output y follows a given function of time $\xi(t)$. That is,

$$e(t) = y(t) - \xi(t) \tag{2.2}$$

converges to zero as t goes to infinity. The index of the DAE (2.1) as a system in (x,u) is one more than the relative degree of (2.1). If there were additional material constraints acting directly on x in (2.1a), then the index could be more than one higher than the relative degree.

Jankowski and van Brussel propose to solve this problem as follows:

- Application of a number of steps of the stabilized structure algorithm to the DAE (2.1) in (x, u). (That is, the path constraint (2.2) is multiplied by a stable polynomial $p(s)$ in the differentiation operator $s = \frac{d}{dt}$. See [8] for a detailed analysis of this method.) The new system

$$\dot{x} = f(x) + G(x)u$$

$$\overline{y} = \widehat{h}(x) + \widehat{J}(x)u$$

has low index (less than or equal 3). Reducing the index to no more than 3 is necessary because the type of numerical integration schemes being used here to solve the DAE, namely BDF methods, need the index to be three or less to be reliable.

- Discretization of the DAE in (X, U).

$$\begin{aligned} \dot{X} &= f(X) + G(X)U && \text{(2.3a)} \\ \widehat{\xi}(t) &= \widehat{h}(X) + \widehat{J}(X)U && \text{(2.3b)} \\ X(t_\rho) &= x(t_\rho) && \text{(2.3c)} \end{aligned}$$

using an implicit discretization scheme. Implicit methods must be used because a DAE is being integrated. The need for fast integration leads to the choice of a first order BDF method (backward Euler),

$$\dot{X}(t+\epsilon) = \frac{X(t+\epsilon) - X(t)}{\epsilon} \ ,$$

where ϵ is the discretization time interval. We let $X_0^\rho = x(t_\rho)$ denote the sampled state vector x at times $t = t_\rho$, $\rho = 1, 2, \cdots$, where $t_{\rho+1} = t_\rho + \delta$ and $\delta > 0$ is the sampling interval. Let $\{X_j^\rho, U_j^\rho\}$ denote the computed estimates for $\{x(t_\rho+\epsilon j), u(t_\rho+\epsilon j)\}$ starting with $X_0^\rho = x(t_\rho)$. The resulting discrete approximation of (2.3) is

$$\begin{aligned} & X_{j+1}^\rho - X_j^\rho = \epsilon\{f(X_{j+1}^\rho + G(X_{j+1}^\rho)U_{j+1}^\rho\} \\ & \widehat{\xi}(t_\rho + j\epsilon) - \widehat{h}(X_{j+1}^\rho) - \widehat{J}(X_{j+1}^\rho)\, U_{j+1}^\rho = 0 \qquad \text{(2.4)} \\ & X_0^\rho = x(t_\rho) \ . \end{aligned}$$

We use uppercase letters for the numerically integrated DAE (2.3) as opposed to the original system (2.1).

- Computation of the numerical solution of (2.4) over the time interval $[t_\rho, t_\rho + \tau]$, where $\tau = k\epsilon$. That is we solve (2.4) from $j = 0$ to $j = k$, where k is an integer to be chosen larger than the number of left out steps in the structure algorithm. Equivalently, k is larger than the index of the stabilized DAE.

- Application of a piecewise constant control $u(t)$ to (2.1)

$$u(t) = \begin{cases} \vdots \\ U_k^\rho \quad \text{for} \quad t_\rho \le t < t_\rho + \delta \\ \text{computed with initial value } X_0^{\rho-1} = x(t_{\rho-1}) \\ U_k^{\rho+1} \quad \text{for} \quad t_{\rho+1} \le t < t_{\rho+1} + \delta \\ \text{computed with initial value } X_0^\rho = x(t_\rho) \\ \vdots \end{cases}$$

Here $\delta \ge \epsilon$ and ϵ is chosen such that $2\delta \ge k\epsilon \ge \delta$.

This procedure is illustrated in Figure 2.1.

Figure 2.1 Representation of the predictive control strategy proposed in [9]. *$x(t)$ represents the state evolution of the controlled plant over time t. At each time t_ρ, the plant is sampled and the vector $x(t_\rho)$ is used as the initial value X_0^ρ for system (2.3). The plot X_j^ρ, U_j^ρ represents the result of the numerical integration of system (2.3) for $j = 0$ to $j = k$. The control value of u over the interval $t_{\rho+1}$ to $t_{\rho+2}$ is chosen to be U_k^ρ. Clearly, to be implementable, the numerical integration of the DAE has to be faster than real-time.*

3. Stability analysis

In [5] we presented a detailed analysis of the predictive control strategy of [9] in the linear case. We considered the linear system

$$\begin{aligned} \dot{x} &= Ax + Bu && \text{(3.1a)} \\ y &= Cx + Du \ . && \text{(3.1b)} \end{aligned}$$

To analyze the properties of the predictive control applied to system (3.1), we made a few simplifying assumptions. The first assumption was that the solution of the DAE (2.3) is constructed without any error, the second assumption was that the time δ between two measurements of the state is very small so that the limiting behavior as δ go to zero is only considered. For the sake of simplicity we assumed that (3.1) represents the partially stabilized system. Without any loss of generality we suppose that (3.1) is in the form

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \\ y &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du \ . \end{aligned} \tag{3.2}$$

where the subspace of x_1 is the largest output nulling (A, B) invariant subspace of (3.1). In [5] we showed that the feedback K obtained from the predictive control strategy of [9], yields the following closed loop dynamics.

$$\begin{aligned} \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} K_1 & 0 \end{bmatrix} &= \begin{bmatrix} \overline{A}_1 & A_2 \\ 0 & A_4 \end{bmatrix} \\ \begin{bmatrix} C_1 & C_2 \end{bmatrix} + DK_1 &= \begin{bmatrix} 0 & \overline{C}_2 \end{bmatrix} \ . \end{aligned}$$

Clearly, the success of this control strategy depends on the stability of $\overline{A}_1$ and A_4. It is well known that the eigenvalues of $\overline{A}_1$ are the zero dynamics of the original system so that $\overline{A}_1$ is a stable matrix provided the original system is minimum phase. Eigenvalues of A_4 on the other hand are not known a priori and can be stable on not. It is exactly these eigenvalues that are "stabilized" by the preliminary feedback introduced in [5].

To be more specific, before the computation of the predictive control, a feedback of the type $u = K_2x_2 + v = K_{pr}x + v$ is applied to the system to place the eigenvalues of A_4. Since the feedback obtained by the predictive controller of [9] does not act on A_4, the controlled system is necessarily stable (provided the system is minimum phase).

Figure 3.1 Representation of the predictive control strategy with preliminary feedback

Of course this analysis is only valid for linear systems, and special care must be taken for its application to nonlinear systems. In particular, the computation of the linear preliminary feedback should be done based on the linearization of the system around a reasonable operating point which is to be updated if necessary (gain scheduling).

4. Example: flexible-joint robot introduced in [1]

To illustrate the importance of the preliminary feedback discussed here we shall examine the flexible-joint robot introduced in [1]. Since we did not have access to the robot the evaluation was performed with computer simulations. As is typical with the path control of flexible joint robots the index of the original DAE in (x, u) is five. If actuator dynamics were included the index would be six [3, 10]. Two stabilized differentiations will be done to make the DAE index three. A couple of other modifications of the basic procedure will be pointed out.

Figure 4.1 *Schematic view of the experimental two-link robot with one flexible joint used in [9].*

Model Parameters	
Parameter	Value
A_1	4.984 kg m^2
A_2	0.247 kg m^2
A_3	0.328 kg m^2
I_{r_1}	1.939 kg m^2
B_{v_1}	1.40 N m s/rad
B_{v_2}	0.20 N m s/rad
F_{c_1}	4.82 N m
F_{c_2}	1.15 N m
ϵ_1	10^{-4} rad
ϵ_2	10^{-4} rad
K_1	1152.6 N m/rad

The dynamic model of the system is

$$\begin{bmatrix} A_1+2A_3\cos q_2 & A_2+2A_3\cos q_2 & 0 \\ A_2+2A_3\cos q_2 & A_2 & 0 \\ 0 & 0 & I_{r_1} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{\theta} \end{bmatrix} =$$

$$\begin{bmatrix} A_3\dot{q}_2(2\dot{q}_1+\dot{q}_2)\sin q_2 \\ -B_{v_2}\dot{q}_2 - x_{f_2} - A_3 q_1^2 \sin q_2 + T_2 \\ -B_{v_1}\dot{\theta} - x_{f_1} + T_1 \end{bmatrix} + K_1 \begin{bmatrix} \theta - q_1 \\ 0 \\ q_1 - \theta \end{bmatrix}$$

$$\begin{aligned} \epsilon_2 \dot{x}_{f_2} &= -|\dot{q}_2| x_{f_2} + F_{c_2}\dot{q}_2 \\ \epsilon_1 \dot{x}_{f_1} &= -|\dot{\theta}| x_{f_1} + F_{c_1}\dot{\theta} \end{aligned} \tag{4.1}$$

where the last two equations model dynamic friction. Control variables are the torques $\{T_1, T_2\}$. State variables are $\{q_1, q_2, \theta, x_{f_1}, x_{f_2}\}$. Following [9], for the predictive control design the output tracking errors $e_1 = (q_1 - q_{d_1})$ and $e_2 = (q_2 - q_{d_2})$ are premultiplied by the polynomial $p^*(s) = p(s)/s$, where $p(s) = s^3 + 3\alpha s^2 + 3\alpha^2 s + \alpha^3$. The integrator is added to avoid the reconstruction of the fourth derivative of q_1. Due to the factor $1/s$ in $p^*(s)$ we have to add the following integrator equations,

$$\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} = \begin{bmatrix} \alpha_1^3(q_1 - q_{d_1}) \\ \alpha_2^3(q_2 - q_{d_2}) \end{bmatrix} \tag{4.2}$$

so that the stabilized error equations become

$$0 = \begin{bmatrix} (\ddot{q}_1 - \ddot{q}_{d_1}) + 3\alpha_1(\dot{q}_1 - \dot{q}_{d_1}) + 3\alpha_1^2(q_1 - q_{d_1}) + w_1 \\ (\ddot{q}_2 - \ddot{q}_{d_2}) + 3\alpha_2(\dot{q}_2 - \dot{q}_{d_2}) + 3\alpha_2^2(q_2 - q_{d_2}) + w_2 \end{bmatrix} \tag{4.3}$$

As long as (4.1),(4.2),(4.3) contain dry friction terms a linear analysis of the DAE is impossible. As proposed in [1], to eliminate the dry friction torques x_{f_i} we add the nonlinear observer

$$\epsilon_i \dot{\widehat{x}}_{f_i} = -|\dot{v}_i| \widehat{x}_{f_i} + F_{c_i}\dot{v}_i \ , \ \ i = 1,2, \tag{4.4}$$

where $v_1 = q_\theta$ and $v_2 = q_2$. Using the observed dry friction torque $\widehat{x}_{f_i}$ we can pre-compensate the effect of dry friction by $u_i = v_i + x_{f_i}$.

Numerical evaluation of the eigenvalues of A_4 for linearizations around different nominal points shows that the eigenvalues stay close to $2.63 \pm j29.58$. As a consequence the predictive control strategy of [9] cannot work based on our linear analysis as given in [4, 5]. This is consistent with the observations in [9], that their control fails for small δ since our analysis was based on the limiting control as $\delta \to 0^+$. By applying the preliminary linear feedback

$$K_{pr} = \begin{bmatrix} 102.3 & -0.07 & -103.0 & 15.97 & -0.10 & -17 & 0 & 0 \\ 16.17 & -0.01 & -16.3 & 2.55 & -0.02 & -2.71 & 0 & 0 \end{bmatrix}$$

obtained by standard LQ techniques [5], the eigenvalues of A_4 are moved to $-2.61 \pm j29.58$. Applying the predictive control to this new system then yields a stable control for all $\delta < \delta_{\max}$.

The necessity of using preliminary feedback is now clear. Even for this example, where by chance, predictive control of [9] works for large δ, the use of preliminary feedback allows us to consider δ as a full design parameter. This is useful because small δ yields small tracking error so that the size of δ should only be lower bounded by limitations of the processor or state estimator. Exactly why the system was stable for large, but not too large, δ is not clear. It might be due to the delay introduced by large δ. However, as noted in [4] and illustrated their with numerical tests, there is a subtle interplay between the control parameters $k, m, \delta, \epsilon, \alpha_i$ that can lead to stability or instability for systems which are not strongly stable or unstable. Since the normalized eigevalues of A_4 in this example are $0.0886 \pm j0.9961$ relatively small changes can alter the stability.

5. Comparative study of the controller with and without preliminary feedback

For comparison purposes we take the same reference trajectory as in [9]. The reference trajectories for both links are

$$q_d(t) = q_{d0} + (q_{df} - q_{d0}) \times \left(\frac{70}{t_f^9} t^9 - \frac{315}{t_f^8} t^8 + \frac{540}{t_f^7} t^7 - \frac{420}{t_f^6} t^6 - \frac{126}{t_f^5} t^5 \right)$$

Here q_{d0} is the initial position, q_{df} is the final position, and t_f is the time required for the slew. For large sampling periods $\delta >> 0.01$ the predictive controller works and preliminary feedback yields no tremendous performance improvement. For smaller δ, without preliminary feedback, the predictive controller destabilizes the controlled system and the use of preliminary feedback is necessary.

Parameters for Reference Function	
Parameter	Value
$q_{d_1 0}$	0 rad
$q_{d_1 f}$	$\frac{\pi}{2}$ rad
t_{f_1}	1 s
$q_{d_2 0}$	$\frac{3\pi}{2}$ rad
$q_{d_2 f}$	0 rad
t_{f_2}	1 s

Controller Parameters	
Parameter	Value
α_1	2
α_2	2
k	12
ϵ	$\frac{\delta}{8}$

Figure 5.1

Predictive Control as proposed in [1] with preliminary feedback: *Tracking error for the first and the second link for a sampling time $\delta = 0.04$ s. Satisfactory errors can be achieved for large δ as long as $\delta < 0.1$ s. The errors are comparable with those obtained in [9].*

Figure 5.2

Predictive Control as proposed in [1] with preliminary feedback: *Tracking error for the first and the second link for a sampling time $\delta = 0.01$ s. The error decreases gradually as δ becomes small.*

Figure 5.3

Predictive Control as proposed in [1]: *Tracking error for the first and the second link for a sampling time $\delta = 0.01$ s. Without Preliminary feedback the system becomes unstable for small sampling periods. A change of the control parameters α_i to smaller values does help.*

6. Conclusion

We have shown that the predictive control strategy proposed in [9] can result in unstable systems. We have given an extension of the proposed control strategy and have shown its application to the flexible-joint robot example used in [9]. The extension maintains all of the potential advantages of the proposed approach but results in enhanced performance. While the preliminary feedback is linear, the final predictive control law is nonlinear and as shown here can be effective on nonlinear systems.

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